$\mathbf{d}^m$

$\mathbf{p}^m$

m

$$\mathbf{x}^{m,n}$$

$\mathbf{x}^{m,n-1}$

$$\mathbf{x}^{m,0}$$

$$\mathbf{X}^{m,N}$$

$\mathbf{f}^n$

$$\underset{\sim}{\mathbf{W}}^n$$

$x_i^{m,n}$

$=$

$$\sum_j W_{ij}^n f_j^n(\mathbf{x}^{m,n-1}),$$

or $\mathbf{x}^{m,n}$

$\underset{\sim}{\mathbf{W}}^n \cdot \mathbf{f}^n(\mathbf{x}^{m,n-1})$ (in vector notation).

Find parameters $\mathbf{p}^m$

$\approx$

$$\mathbf{x}^{m,N}$$

from data $\mathbf{d}^m$

$$\mathbf{x}^{m,0}.$$

$$0 \leq f(x) < f(x') \leq 1$$

$$0 \leq x < x' \leq 1$$

$$df(x)/dx$$

$\mathbf{f}^1$

$$\underset{\sim}{\mathbf{W}}^{N}$$

$\mathbf{p}^m$

$$\mathbf{x}^{m,N}(\mathbf{d}^m, \underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N) = \mathbf{x}^{m,N}.$$

F

$$\min_{\{W_{ij}^n\}} F(\underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N)$$

$$\sum_{m,k} [p_k^m - x_k^{m,N}(\mathbf{d}^m, \underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N)]^2 = \sum_m \|\mathbf{p}^m - \mathbf{x}^{m,N}\|^2.$$

$$\delta \underset{\sim}{\mathbf{W}}^{n}$$

$$\delta F$$

$$\delta F(\underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N)$$

$$-\sum_{m,k}(p_k^m - x_k^{m,N})\delta x_k^{m,N} = -\sum_m (\mathbf{p}^m - \mathbf{x}^{m,N}) \cdot \delta \mathbf{x}^{m,N},$$

where $\delta\mathbf{x}^{m,0}$

0

and $\delta x_i^{m,n}$

$$\sum_j \delta W_{ij}^n \cdot f_j^n(\mathbf{x}^{m,n-1}) + \sum_j W_{ij}^n \sum_k \frac{\partial}{\partial x_k} f_j^n(\mathbf{x}^{m,n-1}) \delta x_k^{m,n-1},$$

or $\delta\mathbf{x}^{m,n}$

$$\delta \underset{\sim}{\mathbf{W}}^{n} \cdot \mathbf{f}^{n}(\mathbf{x}^{m,n-1}) + \underset{\sim}{\mathbf{W}}^{n} \cdot \underset{\sim}{\nabla} \mathbf{f}^{n}(\mathbf{x}^{m,n-1}) \cdot \delta \mathbf{x}^{m,n-1}.$$

$\delta \mathbf{x}^{m,n}$

$$\delta \underset{\sim}{\mathbf{W}}^{n}$$

$$\underset{\sim}{G}^{m,n}$$

$$\delta x_k^{m,N}$$

$$\sum_{n,i,j} G_{kij}^{m,n} \delta W_{ij}^{n}$$

or $\delta\mathbf{x}^{m,N}$

$$\sum_n \underset{\sim}{\mathbf{G}}^{m,n} : \delta \underset{\sim}{\mathbf{W}}^n .$$

$$\delta W_{ij}^{n}$$

$$\sum_m \delta x_i^{m,n} f_j^n(\mathbf{x}^{m,n-1}),$$

or $\delta \underset{\sim}{\mathbf{W}}^n$

$$\sum_m \delta \mathbf{x}^{m,n} [\mathbf{f}^n(\mathbf{x}^{m,n-1})]^* \quad \text{(outer product)},$$

where $\delta\mathbf{x}^{m,N}$

$$-(\mathbf{p}^m - \mathbf{x}^{m,N})\delta F$$

and $\delta x_k^{m,n-1}$

$$\sum_j \frac{\partial}{\partial x_k} f_j^n(\mathbf{x}^{m,n-1}) \sum_i W_{ij}^n \delta x_i^{m,n},$$

or $\delta\mathbf{x}^{m,n-1}$

$$\left[\underset{\sim}{\nabla}\mathbf{f}^n(\mathbf{x}^{m,n-1})\right]^* \cdot (\underset{\sim}{\mathbf{W}}^n)^* \cdot \delta\mathbf{x}^{m,n}.$$

$$\sum_{m,k} G_{kij}^{m,n} \delta x_k^{m,N},$$

$$(\underset{\sim}{\mathbf{G}}^{m,n})^* \cdot \delta \mathbf{x}^{m,N}.$$

$$\Delta W_{ij}^{n} = \sum_{m,k} G_{kij}^{m,n}[p_k^m - x_k^{m,N}(\mathbf{d}^m, \underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N)],$$

$$\Delta \underset{\sim}{\mathbf{W}}^{n} = (\underset{\sim}{\mathbf{G}}^{m,n})^{*} \cdot (\mathbf{p}^{m} - \mathbf{x}^{m,N})$$

$$\underset{\sim}{\mathbf{W}}^n + \epsilon \Delta \underset{\sim}{\mathbf{W}}^n$$

ϵ

$$\Delta \underset{\sim}{\mathbf{W}}^{n}$$

α

$$\sum_{m,k} [p_k^m - x_k^{m,N}(\mathbf{d}^m, \underset{\sim}{\mathbf{W}}^1 + \alpha \Delta \underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N + \alpha \Delta \underset{\sim}{\mathbf{W}}^N)]^2$$

$$\underset{\sim}{\mathbf{W}}^n + \alpha \Delta \underset{\sim}{\mathbf{W}}^n$$

$$\Delta \mathbf{x}^{m,N} = \sum_n \underset{\sim}{\mathbf{G}}^{m,n} : \Delta \underset{\sim}{\mathbf{W}}^n$$

$$\sum_{m,k} [p_k^m - \alpha \Delta x_k^{m,N}]^2$$

$$\alpha = \left(\sum_{m,k} p_k^m \Delta x_k^{m,N}\right) / \sum_{m,k} \Delta x_k^{m,N} \Delta x_k^{m,N}.$$

$$\{\underset{\sim}{\mathbf{w}}^1, \ldots, \underset{\sim}{\mathbf{w}}^N\}$$

$$\sum_{m,k} [p_k^m - x_k^{m,N}(\mathbf{d}^m, \underset{\sim}{\mathbf{W}}^1, \ldots, \underset{\sim}{\mathbf{W}}^N) - \sum_{n,i,j} G_{kij}^{m,n} w_{ij}^n]^2$$

$$\underset{\sim}{\mathbf{W}}^n + \underset{\sim}{\mathbf{w}}^n$$

$$\underset{\sim}{\mathbf{w}}^{n}$$