Tieman's conversion of common-midpoint slant stacks to common-source

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Hans Tieman spoke to the Stanford Exploration Project on Jan 23, 1996 about depth imaging with slant slacks. Among his techniques is a clever method of converting slant stacks of midpoint gathers into equivalent slant stacks of source gathers. A source gather best represents a physical experiment that can be modeled easily by wave-equation methods. Midpoint gathers, however, better include the coherence of steep reflections and better avoid aliasing. A conversion takes advantage of the best of both domains.

The data

Seismic amplitudes $d(s, r, t)$ are recorded over time t as a function of surface horizontal positions for source s and receiver r. Although these positions are one-dimensional we must also be prepared to think of them as vectors indicating a surface position. The time coordinate is sampled evenly and densely enough that we can think of it as continuous.

For a given source, we have a limited range of receivers (perhaps 3–5 kilometers), and vice versa. Receiver positions are often sampled two or four times as densely as source positions. In marine data, both are relatively evenly sampled, but a spatial Fourier transform must pay attention to aliasing or edge effects from the short span. Land data will be much more arbitrarily sampled.

Define the coordinates of full offset $h \equiv r - s$ and midpoint $y \equiv (s + r)/2$. Resorted data can be written as $d(s = y - h/2, r = y + h/2, t)$. The well-sampled midpoint coordinate covers the entire span of the survey.

A conventional slant stack

Slant stacks are commonly applied to unsorted data, one shot at a time. This form is well suited to deconvolution of multiple reflections from flat reflectors. Such multiple reflections are periodic at zero-offset, but not at a single finite offset h.

A slant stack attempts to describe our recorded data as a sum of dipping lines. A dip p_s will measure the slope of time with offset holding a source position constant.

$$p_s \equiv \left. \frac{\partial t}{\partial h} \right|_s \tag{1}$$

With ideal sampling and infinite offsets, this equation would describe a plane-wave source on the surface. A plane wave reflecting from flat reflectors would produce periodic multiples at any p_s. Predictive deconvolutions can detect this periodicity and remove multiple reflections.

The simplest slant-stack sums data over all lines within a feasible range of dips. Let τ_s be the intersection at zero offset of our imaginary plane wave in the shot gather.

$$S(s, p_s, \tau_s) \equiv \int d(s, r = s + h, t = \tau_s + p_s h) dh \tag{2}$$

In practice the integral over offset h must be a discrete sum with a limited range of offsets.

The inverse of this transform looks much like another slant stack, with some adjustments of the spectrum. Papers are readily available to explain this inverse. I will concentrate instead on the conversion of one type of slant stack to another.

The Fourier version

Because the time axis is well sampled and unaliased, we can safely Fourier transform the data between time t and frequency f:

$$d(s, r, t) \equiv \int \exp(i2\pi f t) \tilde{d}(s, r, f) df. \tag{3}$$

Tildes will indicate Fourier transforms. Transform the slant stack from τ_s to its frequency f_s:

$$\tilde{S}(s, p_s, f_s) = \int \exp(-i2\pi f_s \tau_s) S(s, p_s, \tau_s) d\tau_s. \tag{4}$$

The slant stack simplifies numerically. Substitute the transform (3) into the slant stack (2), then take the transform (4) of both sides of the equation:

$$\tilde{S}(s, p_s, f_s) = \int\int\int \exp(-i2\pi f_s \tau_s) \exp(i2\pi f(\tau_s + p_s h)) \tilde{d}(s, r = s + h, f) dh df d\tau_s. \tag{5}$$

Rearranging terms, we reduce integrals

$$\begin{aligned}
\tilde{S}(s, p_s, f_s) &= \int\int \{\int \exp[-i2\pi(f_s - f)\tau_s] d\tau_s\} \exp(i2\pi f p_s h) \tilde{d}(s, r = s + h, f) dh df \\
&= \int\int \{\delta(f_s - f)\} \exp(i2\pi f p_s h) \tilde{d}(s, r = s + h, f) dh df \\
&= \int \exp(i2\pi f_s p_s h) \tilde{d}(s, r = s + h, f = f_s) dh. \qquad (6)
\end{aligned}$$

The second step uses the Fourier transform of a delta function $\delta(f) = \int \exp(-i2\pi f t) dt$. The third uses the behavior of a delta function in an integral $\int g(f)\delta(f - f_0) df = g(f_0)$.

The midpoint gather

We should prefer a transform of a single source gather because these gathers correspond to a physical experiment that can be modeled easily by wave-equation methods. Unfortunately, reflections from dipping layers and point scatters may have a very complicated expression in a source gather. We may be obliged to use many dips to capture their coherence. Worse, many reflections will have minimum times at finite offset, and a slant stack will alias some of their energy.

If the data are first sorted by midpoint y and half-offset h, then reflections from dipping lines and from points will still remain symmetric about zero offset. A slant stack of a midpoint gather will better capture the coherence of the reflections:

$$Y(y, p_y, \tau_y) \equiv \int d(s = y - h/2, r = y + h/2, t = \tau_y + p_y h) dh \tag{7}$$

where

$$p_y = \left.\frac{\partial t}{\partial h}\right|_y . \tag{8}$$

The Fourier version of a common-midpoint slant stack can be derived exactly as before. Let f_y be the Fourier frequency of τ_y:

$$\tilde{Y}(y, p_y, f_y) = \int \exp(i2\pi f_y p_y h) \tilde{d}(s = y - h/2, r = y + h/2, f = f_y) dh. \tag{9}$$

Unfortunately, this slant stack does not correspond to any single seismic experiment, and wave-equation modeling is much more awkward.

Conversion of midpoint to source gather

Fortunately, we can convert this common-midpoint transform (9) into an equivalent common-source transform (6). Let us make two additional Fourier transforms over spatial dimensions of s and y for the spatial frequencies k_s and k_y:

$$\begin{aligned} \tilde{\tilde{S}}(k_s, p_s, f_s) &= \int \exp(-i2\pi k_s s) \tilde{S}(s, p_s, f_s) ds \\ &= \int\int \exp(-i2\pi k_s s) \exp(i2\pi f_s p_s h) \tilde{d}(s, r = s + h, f = f_s) dh ds \end{aligned} \tag{10}$$

and

$$\begin{aligned} &\tilde{\tilde{Y}} \quad (k_y, p_y, f_y) = \int \exp(-i2\pi k_y y) \tilde{Y}(y, p_y, f_y) dy \\ &= \int\int \exp(-i2\pi k_y y) \exp(i2\pi f_y p_y h) \tilde{d}(s = y - h/2, r = y + h/2, f = f_y) dh dy. \end{aligned} \tag{11}$$

To place the second integral (11) in the form of the first (10), we should change the variables of integration from h and y to h and s. (The Jacobian of this transformation is $\partial(h, y)/\partial(h, s) = 1$.) Substituting $y = s + h/2$ we get

$$\begin{aligned} &\tilde{\tilde{Y}} \quad (k_y, p_y, f_y) \\ &= \int\int \exp[-i2\pi k_y (s + h/2)] \exp(i2\pi f_y p_y h) \tilde{d}(s, r = s + h, f = f_y) ds dy \\ &= \int\int \exp(-i2\pi k_y s) \exp[i2\pi f_y h(p_y - k_y/2f_y)] \tilde{d}(s, r = s + h, f = f_y) ds dy \\ &= \tilde{\tilde{S}}(k_s = k_y, p_s = p_y - k_y/2f_y, f_s = f_y). \end{aligned} \tag{12}$$

Thus, a two-dimensional stretch of the midpoint-gather transform becomes equivalent to the source-gather transform. For a given dip over offset in a midpoint gather p_y, we can identify a dip over midpoint

$$-k_y/f_y = \left.\frac{\partial \tau_y}{\partial y}\right|_{p_y} = \left.\frac{\partial t}{\partial y}\right|_h . \tag{13}$$

The adjustment of $p_s = p_y - k_y/2f_y$ subtracts half of this midpoint dip from the offset dip. With a careful application of the chain rule, and carefully distinguishing partial derivatives, we could arrive at the same result

$$\left.\frac{\partial t}{\partial h}\right|_s = \left.\frac{\partial t}{\partial h}\right|_y + \frac{1}{2}\left.\frac{\partial t}{\partial y}\right|_h . \tag{14}$$