$$w(t)$$

$$\tilde{w}(s)$$

$$\tilde{w}(s)$$

=

$\int e^{-i2\pi st} w(t)dt$ and

$w(t)$

$$\int e^{i2\pi st}\tilde{w}(s)ds.$$

$$C_w = \int \frac{1}{|s|} \tilde{w}(s) ds.$$

$$\tilde{w}(s)/|s|^{\epsilon}$$

$\rightarrow 0$

$s \to 0$

$\epsilon > 0$

$$w(t) = \cos(2\pi t)(e^{-\pi t^2/c^2})$$

C

$$\int w(ut)du = C_w \cdot \delta(t).$$

$t = 0$

Proof : $\int w(ut)du = \int \left\{ \int \frac{1}{|u|} \tilde{w}(\frac{s}{u}) e^{i2\pi st} ds \right\} du \ldots$

(s, u)

$$(s' = s, u' = s/u)$$

$$\partial(s,u)/\partial(s',u') = |s'/u'^2|$$

$$\int w(ut)du$$

$$\int \int \left| \frac{u'}{s'} \right| \tilde{w}(u') e^{i2\pi s' t} \left| \frac{s'}{u'^2} \right| ds' du'$$

$$\int \int \frac{1}{|u'|} \tilde{w}(u') e^{i2\pi s' t} ds' du'$$

$$\left\{\int \frac{1}{|u'|}\tilde{w}(u')du'\right\} \cdot \left\{\int e^{i2\pi s't}ds'\right\}$$

$$C_w \cdot \delta(t).$$

$$w(t) = w(-t).$$

$$F(u, a)$$

L^2

$f(t)$

$$F(u,a) = \int w[u(a-t)]f(t)dt.$$

a

u

2^u

$$u^{-1}$$

$$f(t) = C_w^{-1} \int F(u, a = t) du.$$

$$\textbf{Proof}: \; C_w^{-1} \int F(u, a = t) du$$

$$C_w^{-1} \int \left\{ \int w[u(t-t')] f(t') dt' \right\} du$$

$$C_w^{-1} \int \left\{ \int w[u(t-t')]du \right\} f(t')dt'$$

$$C_w^{-1} \int C_w \delta(t - t') f(t') dt'$$

$f(t).$

$$g(\tau) = f[t(\tau)].$$

$t(\tau)$

t

$\mathcal{T}$

$t(\tau)$

$\approx$

$t(\tau_0) + (\tau - \tau_0) \cdot \frac{dt}{d\tau}(\tau_0)$ and

$\tau(t)$

$$\tau_0 + [t - t(\tau_0)] \cdot \left[\frac{dt}{d\tau}(\tau_0)\right]^{-1} .$$

$$g(\tau)$$

$$G(v, b) = \int w[v(b - \tau)]g(\tau)d\tau \quad \text{and}$$

$$g(\tau) = C_w^{-1} \int G(v, b = \tau) dv.$$

$$G(v, b)$$

$$G(v,b) \approx \left|\frac{dt}{d\tau}(b)\right|^{-1} \quad F\left\{u = v \cdot \left[\frac{dt}{d\tau}(b)\right]^{-1},\ a = t(b)\right\}.$$

Proof : $G(v, b)$

$$\int w[v(b-\tau)] \cdot f[t(\tau)] d\tau$$

$$\int w[v(b-\tau)] \cdot f\left[t(b) + (\tau - b) \cdot \frac{dt}{d\tau}(b)\right] d\tau \ldots$$

t'

≡

$t(b) + (\tau - b) \cdot \frac{dt}{d\tau}(b)$ and

𝓣

$$b + [t' - t(b)] \cdot \left[\frac{dt}{d\tau}(b) \right]^{-1} .$$

$$G(v, b)$$

$$\int w \left\{ v \cdot [t(b) - t'] \cdot \left[\frac{dt}{d\tau}(b) \right]^{-1} \right\} \cdot f(t') \cdot \left| \frac{dt}{d\tau}(b) \right|^{-1} \; dt'$$

$$\left|\frac{dt}{d\tau}(b)\right|^{-1}\int w\left\{v\left[\frac{dt}{d\tau}(b)\right]^{-1}\cdot[t(b)-t']\right\}\cdot f(t')dt'$$

$$\left|\frac{dt}{d\tau}(b)\right|^{-1} \quad F\left\{u = v \cdot \left[\frac{dt}{d\tau}(b)\right]^{-1},\ a = t(b)\right\}.$$